Generalized Loopy 2U: A New Algorithm for Approximate Inference in Credal Networks

Alessandro Antonucci, Marco Zaffalon, Sun Yi, Cassio de Campos

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Imprecise probabilities (Walley, 1991)

Models of uncertainty about the state of a categorical variable X

- A probability mass function $P(X)$
- More generally, a *closed convex set* of probability mass functions $K(X)$ This is a *credal set* (Levi, 1980)
- Complete ignorance? A *vacuous* credal set $K_0(X)$
- Lower (and upper) expectation

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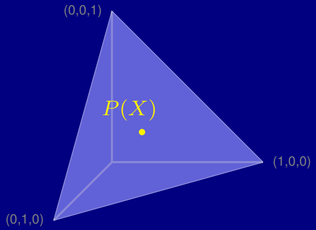

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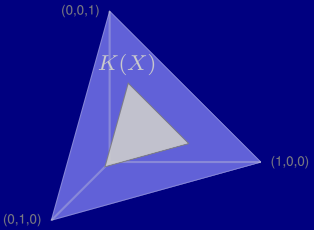

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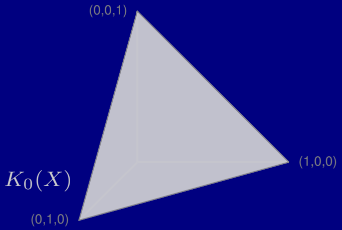

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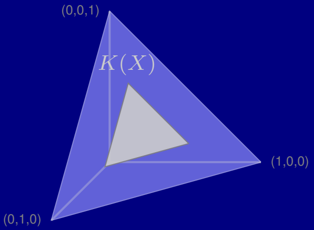

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- Nodes binarization
  - State of a variable as a joint state of a number of "bits"
    $X = x \iff (\tilde{X}^1 = \tilde{x}^1) \wedge (\tilde{X}^2 = \tilde{x}^2) \wedge \dots$
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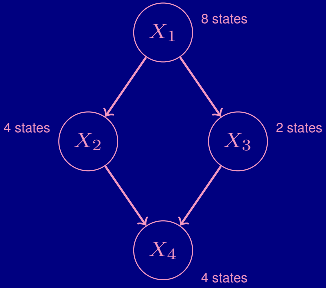

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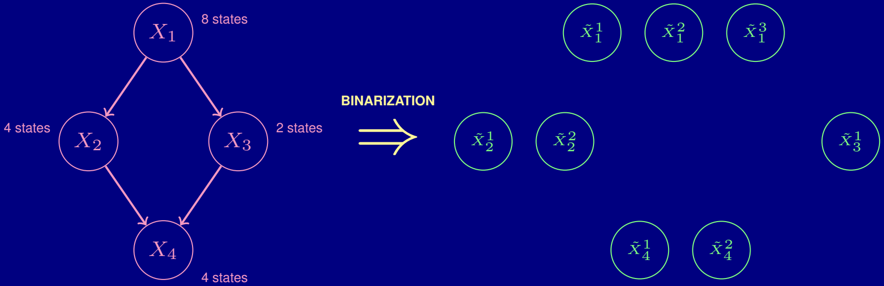

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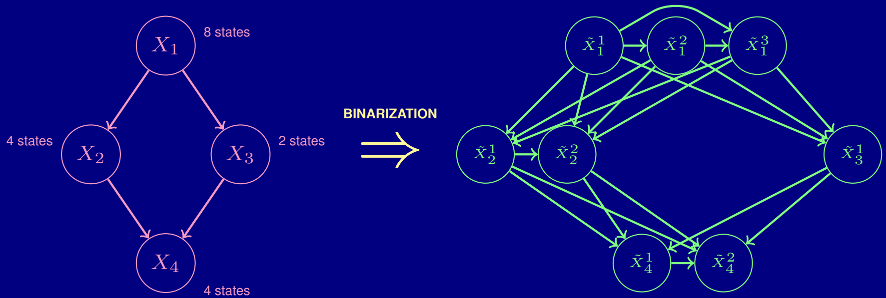

Binarization (probabilities)

- Bayesian nets
  - All the conditional mass function $P(\tilde{X}_i^j|\mathrm{pa}(\tilde{X}_i^j))$ of the bits of X_i can be computed from the conditional mass function $P(X_i|\mathrm{pa}(X_i))$

$$P(\tilde{x}_i^j|\mathrm{pa}(\tilde{X}_i^j)) \propto \sum'_{x_i} P(x_i|\mathrm{pa}(X_i))$$

- Credal nets
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Binarization (probabilities)

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Binarization (results)

BNs binarization is exact

- Let $P(\mathbf{X})$ be the joint probability mass function of a BN,
- and $\tilde{P}(\tilde{\mathbf{X}})$ the corresponding p.m.f. on the binarized BN.
- Then, $P(\mathbf{X}) = \tilde{P}(\tilde{\mathbf{X}})$.

CNs binarization is an outer approximation

- Let $K(\mathbf{X})$ be the strong extension of a CN,
- and $\tilde{K}(\tilde{\mathbf{X}})$ the strong extension of its binarization.
- Then, $K(\mathbf{X}) \subseteq \tilde{K}(\tilde{\mathbf{X}})$.

We can do better!
But another transformation should be applied
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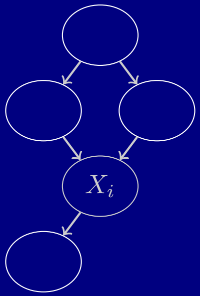
Decision-theoretic specification of CNs

(Antonucci & Zaffalon, PGM '06/IJAR 2008)

- For each $i = 1, \ldots, n$, add a node T_i, parent of X_i, between X_i and $\mathrm{pa}(X_i)$
- Decision nodes $\{T_i\}_{i=1}^n$ indexing the possible specifications of each mass function given the values of the parents
- Decision nodes can be regarded as chance nodes with a "vacuous" specification of the relative credal set
- An equivalent credal net is obtained!



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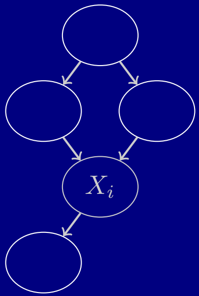

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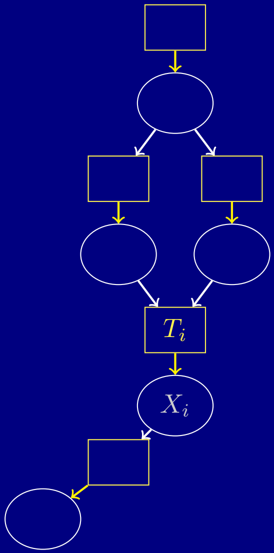

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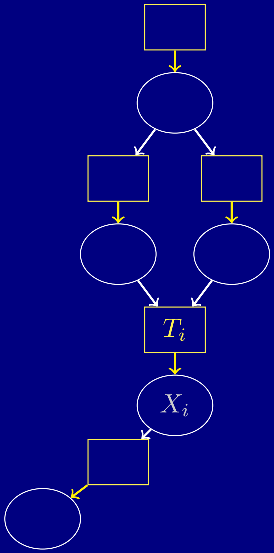

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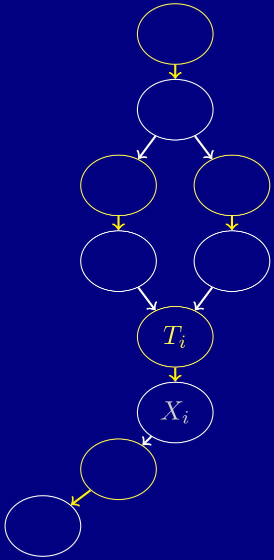

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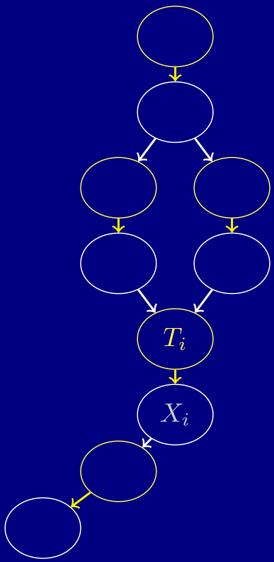
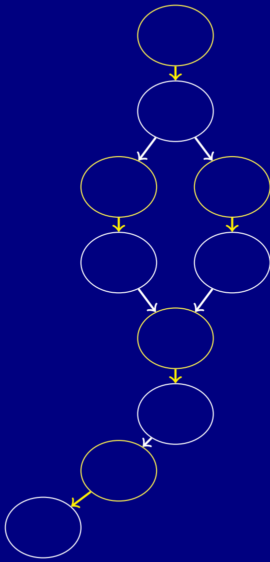


Binarization of DT-specified CNs

- Binarization can be implemented locally
- After DT specification, the nodes are either precise or vacuous
  - For precise specifications binarization is exact (result for BNs)
  - Also for vacuous specifications binarization can be proved to be exact
- Binarization of DT-specified CNs is exact!
- But any CN can be DT-specified
- Any CN admits an exact binarization



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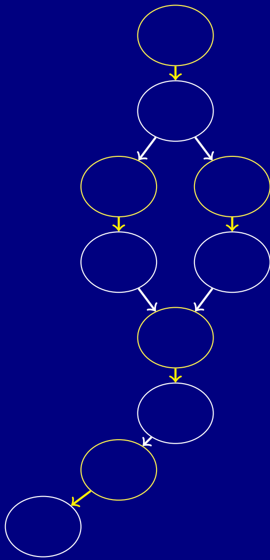

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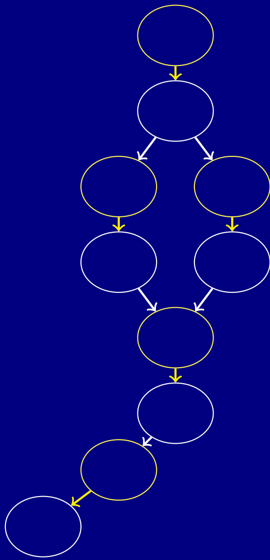

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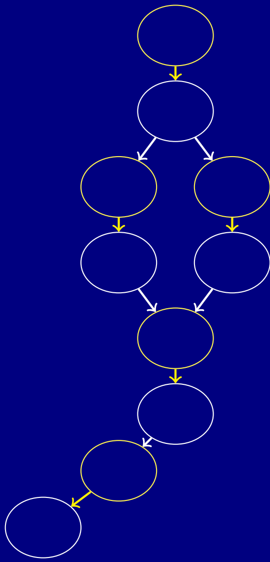

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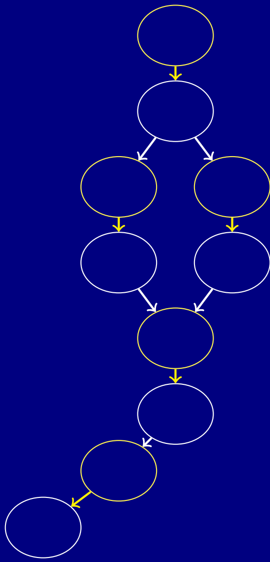

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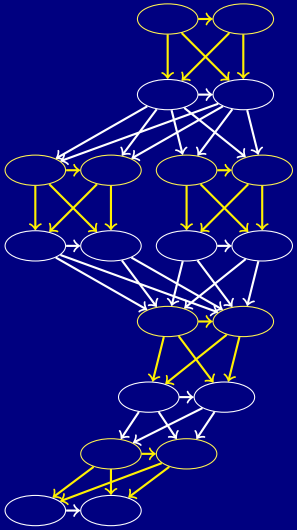

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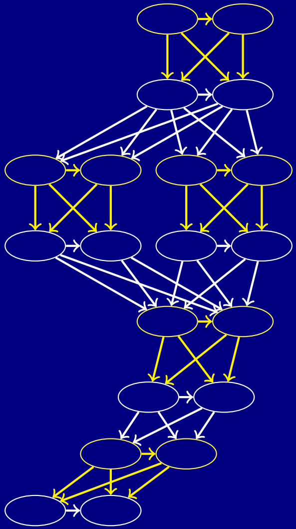

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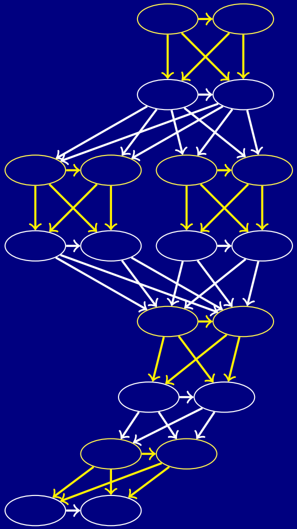

Generalized loopy 2U (GL2U)

- Multi-connected (non-binary) CNs?
- Binarization + L2U (twofold approx)
- DT + Bin + L2U = GL2U is better!
  - Approx only because of loopy!
  - State-of-the-art updating algorithm for CNs updating
  - Good accuracy and scalability $O(e^{\text{indegree}_{\max}})$ is better than $O(e^{\text{treewidth}})$

	Loc Search	GL2U	Bin+L2U
Multi-10	1.89	1.40	1.81
Multi-10	1.95	1.07	3.38
Multi-10	1.20	1.75	3.08
Multi-10	0.27	1.25	2.22
Multi-10	2.34	1.89	6.93
Multi-25	2.31	1.60	1.84
Multi-25	2.48	2.04	3.03
Polyt-50	1.12	1.93	2.89
Polyt-50	1.45	2.21	3.92
Insurance	0.55	1.17	1.75
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Insurance	0.55	1.17	1.75
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Generalized loopy 2U (GL2U)

- Multi-connected (non-binary) CNs?
- Binarization + L2U (twofold approx)
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  - Approx only because of loopy!
  - State-of-the-art updating algorithm for CNs updating
  - Good accuracy and scalability $O(e^{\text{indegree}_{\text{max}}})$ is better than $O(e^{\text{treewidth}})$

	Loc Search	GL2U	Bin+L2U
Multi-10	1.89	**1.40**	1.81
Multi-10	1.95	**1.07**	3.38
Multi-10	**1.20**	1.75	3.08
Multi-10	**0.27**	1.25	2.22
Multi-10	2.34	**1.89**	6.93
Multi-25	2.31	**1.60**	1.84
Multi-25	2.48	**2.04**	3.03
Polyt-50	**1.12**	1.93	2.89
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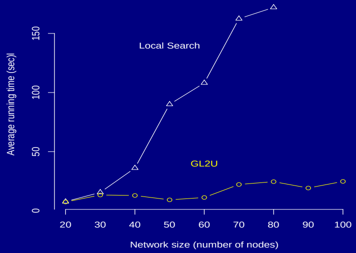

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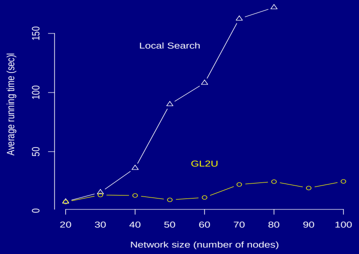

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- The algorithm of choice for very large nets?
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