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Intelligent Systems Group

An empirical analysis of loopy belief propagation in three topologies: grids, small-world networks and random graphs

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Outline

- Motivation
- Belief Propagation
- FlexLBP
- Experiments
- Conclusions

Motivation

- Belief propagation
  - Commonly used for inference
  - Based on message-passing
  - Exact algorithm in trees
  - Good results in structures with cycles
    - Depends on different parameters
    - What about topology?

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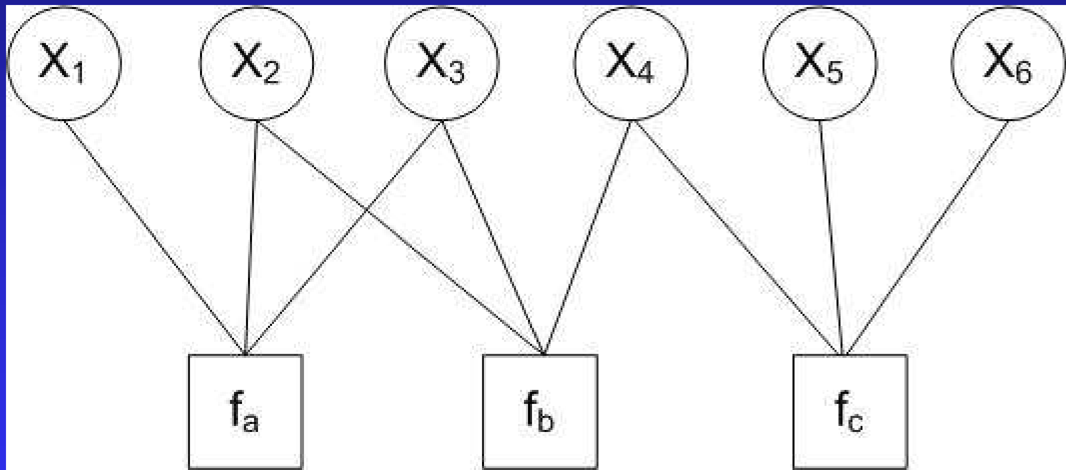
Belief Propagation

- When used for inference
  - Marginal distribution
  - Most probable state
- Based on message-passing
  - Messages are calculated by each node and sent to other nodes

Belief Propagation

- Can be applied to different models
  - We focus on factor graphs

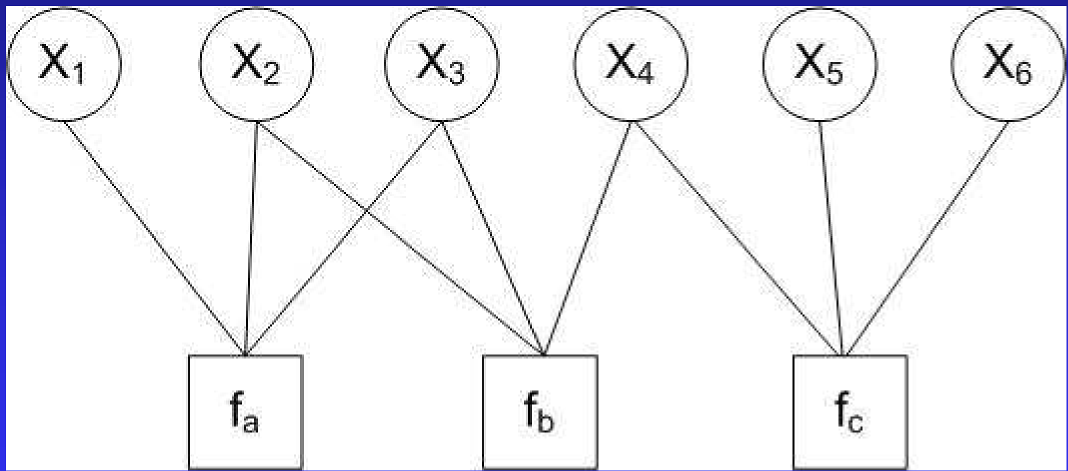


- Joint probability distribution

$$p(x_1,\dots,x_6)=\frac{1}{Z}f_a(x_1,x_2,x_3)f_b(x_2,x_3,x_4)f_c(x_4,x_5,x_6)$$

Belief Propagation

- Each node receives, updates, and sends messages:

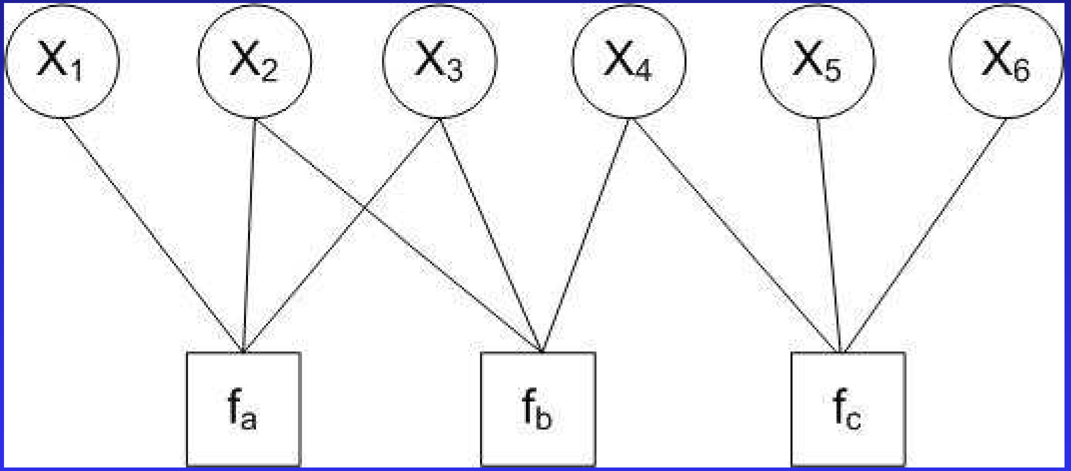


- Messages from variables to factors

$$n_{i\to a}(x_i) = \prod_{c\in N(i)\backslash a} m_{c\to i}(x_i)$$

Belief Propagation

- Each node receives, updates, and sends messages:

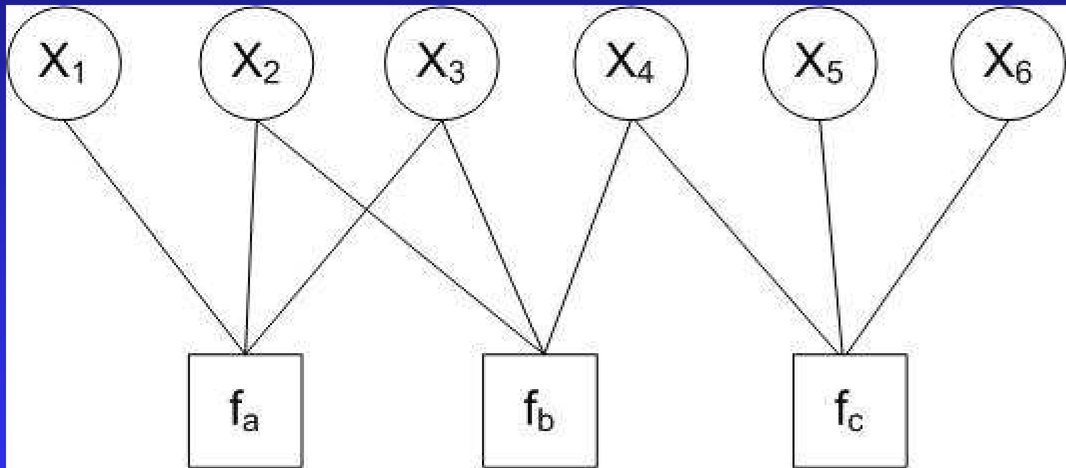


- Messages from variables to factors

$$n_{3\to b}(x_3) = m_{a\to 3}(x_3)$$

Belief Propagation

- Each node receives, updates, and sends messages:

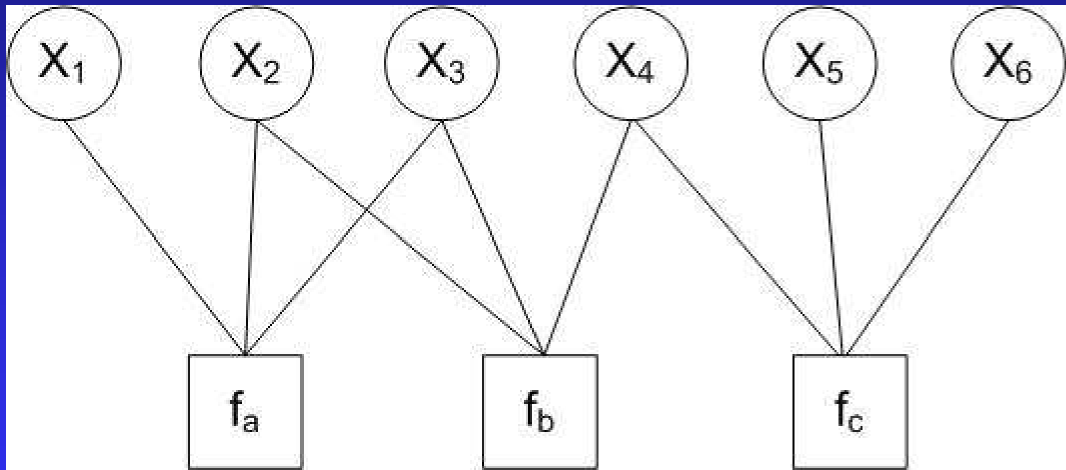


- Messages from factors to variables (inference)

$$m_{a\rightarrow i}(x_i) = \sum_{\chi_a \setminus x_i} f_a(\chi_a) \prod_{j \in N(a)\setminus i} n_{j\rightarrow a}(x_j)$$

Belief Propagation

- Each node receives, updates, and sends messages:

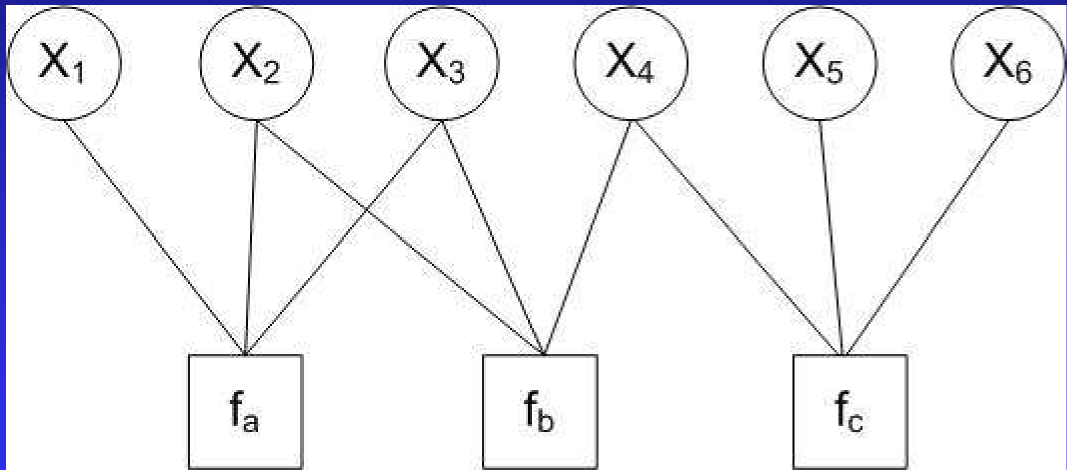


- Messages from factors to variables (inference)

$$m_{b\to 2}(x_2) = \sum_{\chi_b \setminus x_2} f_b(x_2, x_3, x_4) n_{3\to b}(x_3) n_{4\to b}(x_4)$$

Belief Propagation

- Each node receives, updates, and sends messages:



- Messages from factors to variables (optimization)

$$m_{a \to i}(x_i) = \arg\max_{\chi_a \setminus x_i} \{ f_a(\chi_a) \prod_{j \in N(a) \setminus i} n_{j \to a}(x_j) \}$$

Belief Propagation

- Analysis of Loopy Belief Propagation
  - How and when are the messages sent?
    - Scheduling policies
  - When does the algorithm finish?
    - Stopping criteria
  - Which parameters are needed?
    - Initial settings

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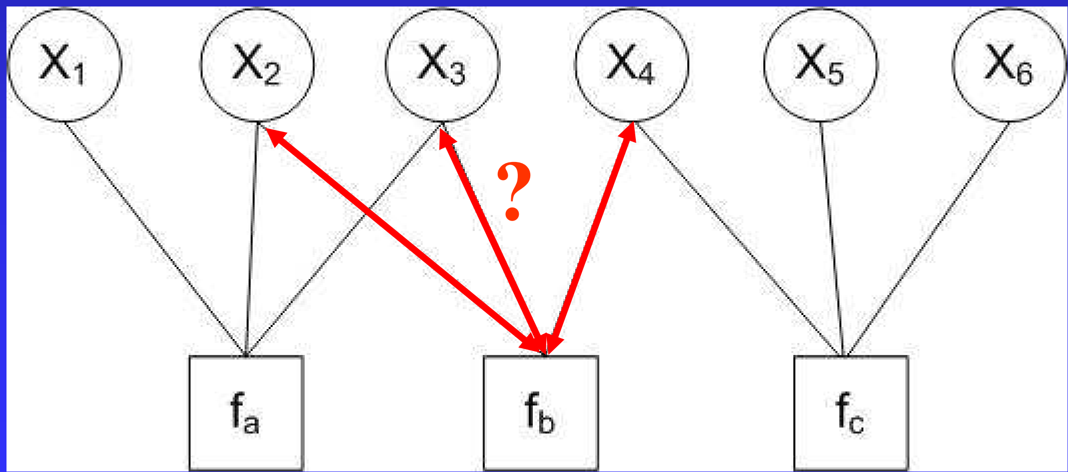
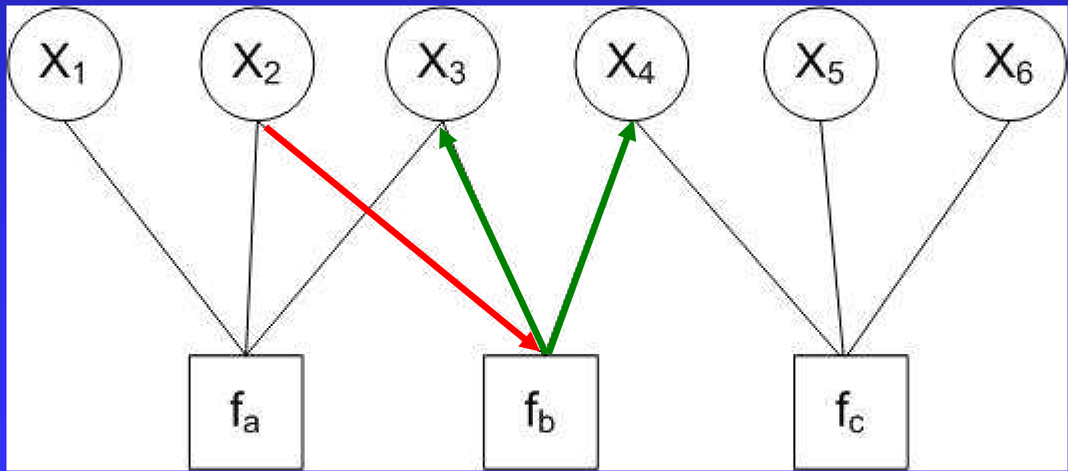
FlexLBP

- Follows a distributed scheme
- Main characteristics:
  - Scheduling policies
    - Number-based or Set-based



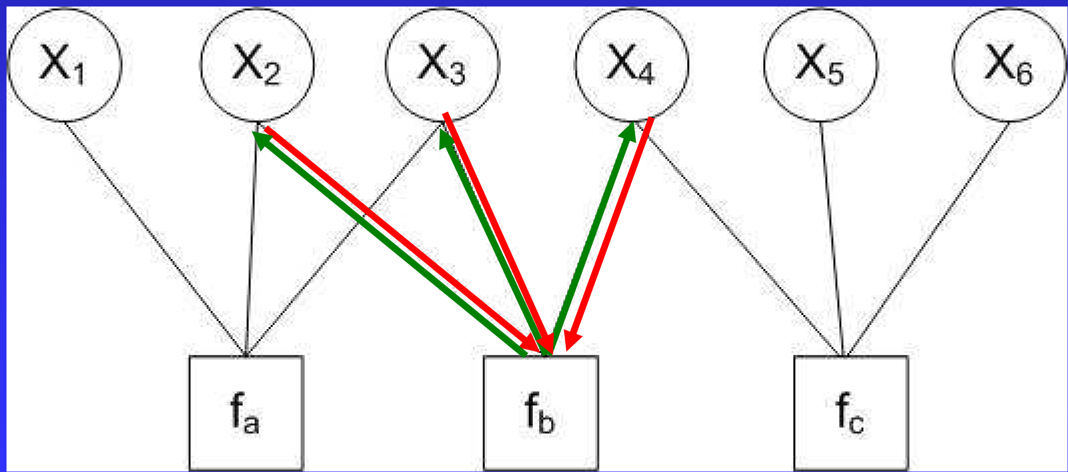
FlexLBP

- Scheduling policies
  - Number-based
    - Send messages when m messages are received



FlexLBP

- Scheduling policies
  - Set-based
  - Recv set $\{X_2,X_3,X_4\}$, Send set $\{X_2,X_3,X_4\}$



FlexLBP

- Stopping criteria
  - Calculated by each node
  - Same message value or sequence
  - A given number of iterations
- Algorithm stops when all the nodes have finished

FlexLBP

- Initial settings
  - Threshold
  - Maximum number of iterations,
  - Number of comparisons needed,
  - Initial values of the messages,
  - Initial ordering,
  - Fixed nodes

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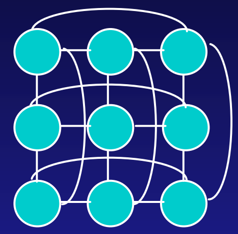
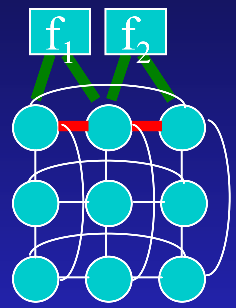
Experiments

- FlexLBP on three different topologies
  - Grids
  - Small-world networks
  - Random graphs
- Two types of experiments
  - Arc rewiring
  - Add shortcuts

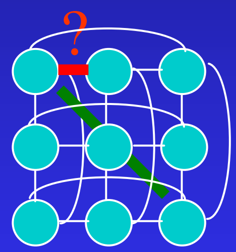
Experiments. Arc Rewiring



Starting from a grid (7x7)



10 sets of random functions for the grid



Each arc will be changed with probability P

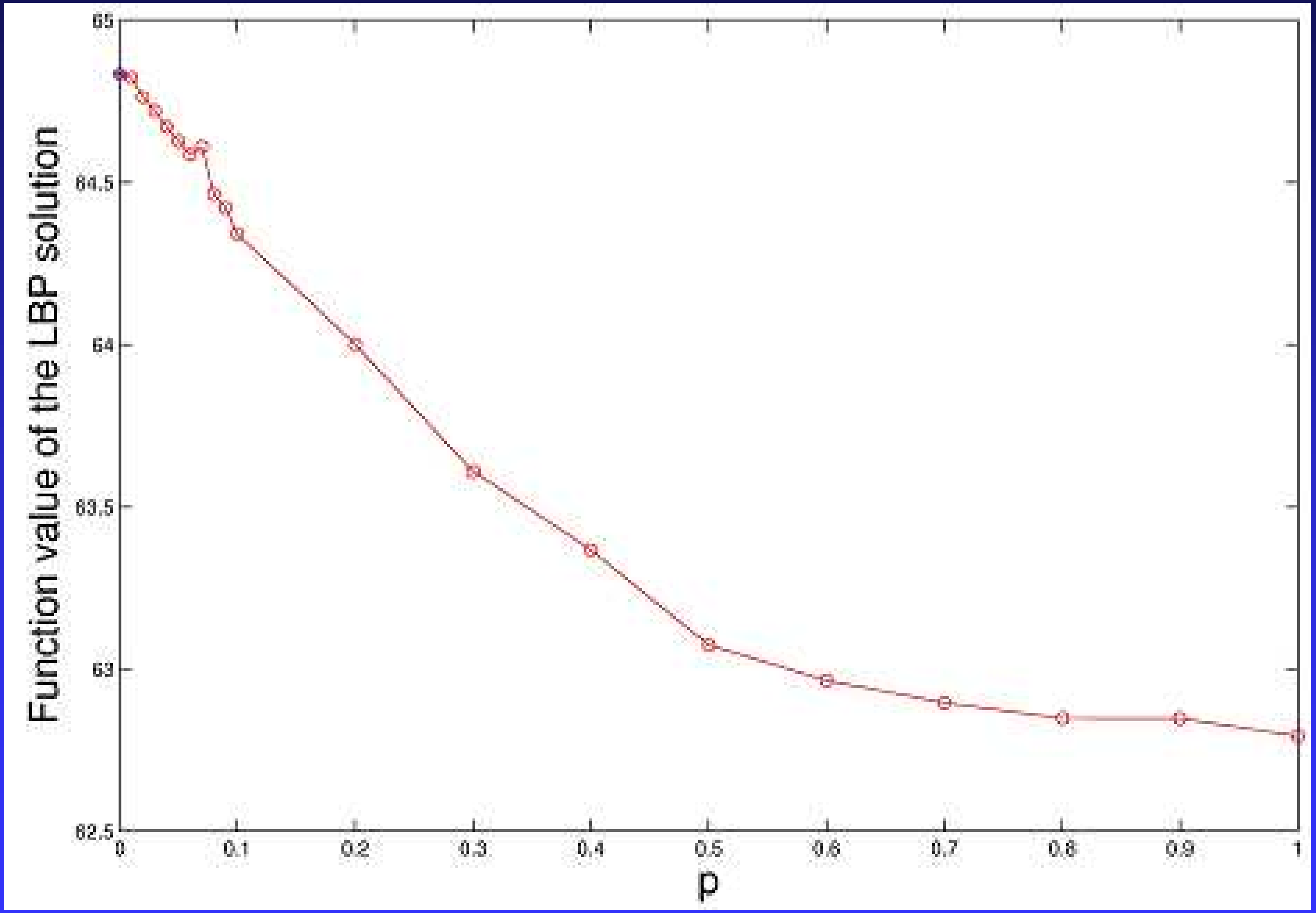
P={0.01, 0.02, .., 0.1, 0.2, .., 0.9, 1.0}

100 instances for each P value

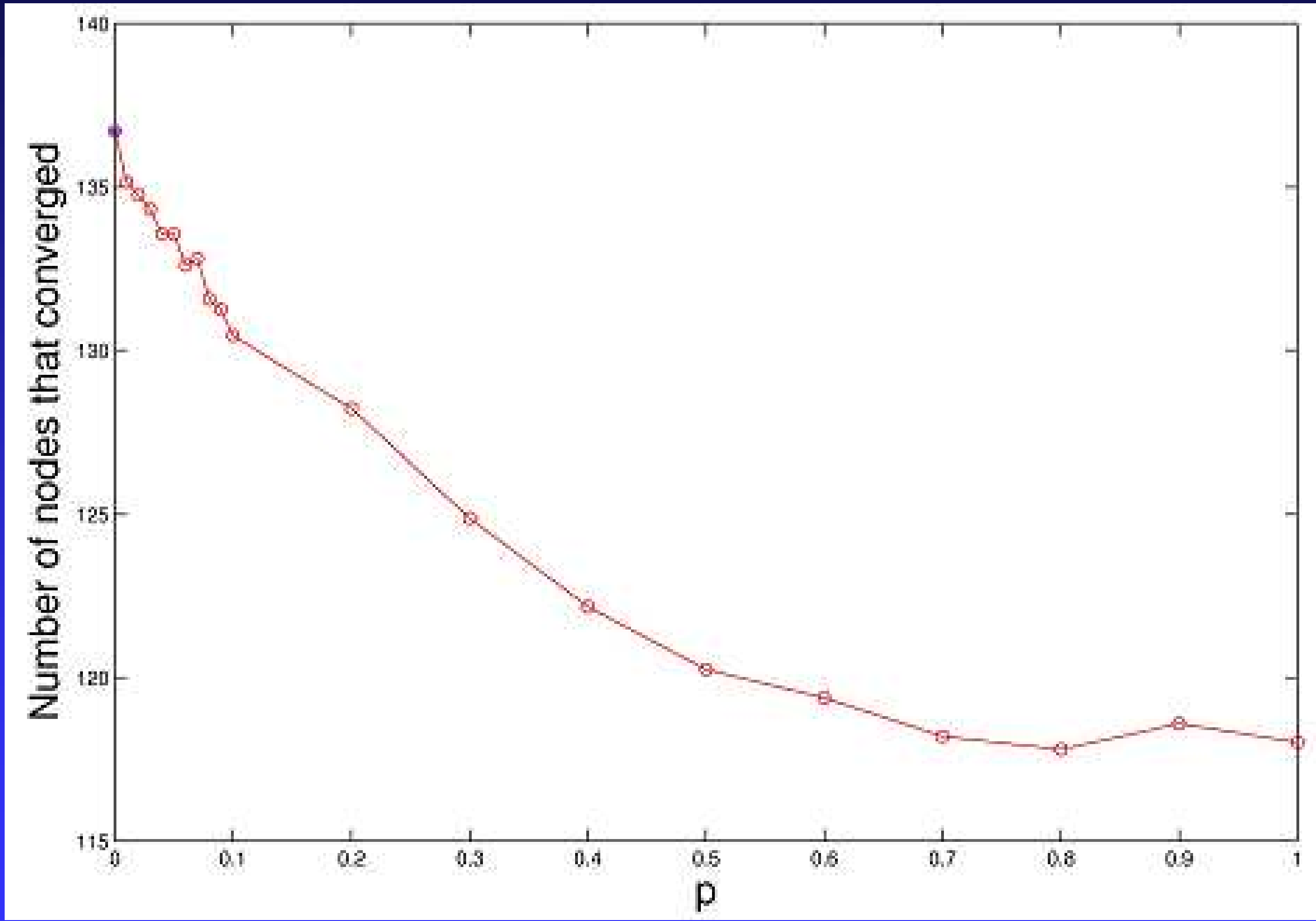
Sum-product and Max-product

Total: 38.000 runs

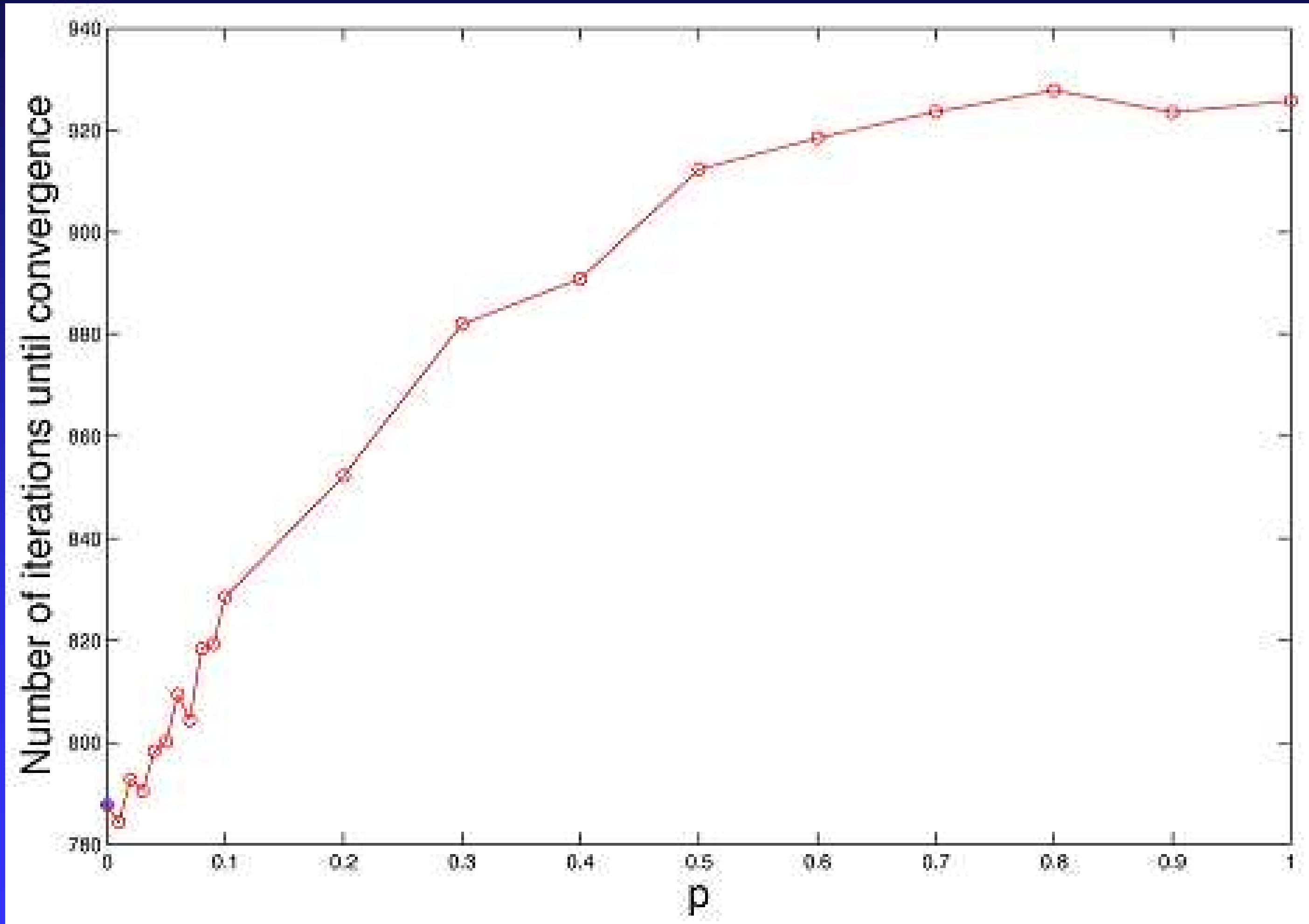
Experiments. Arc rewiring (max)



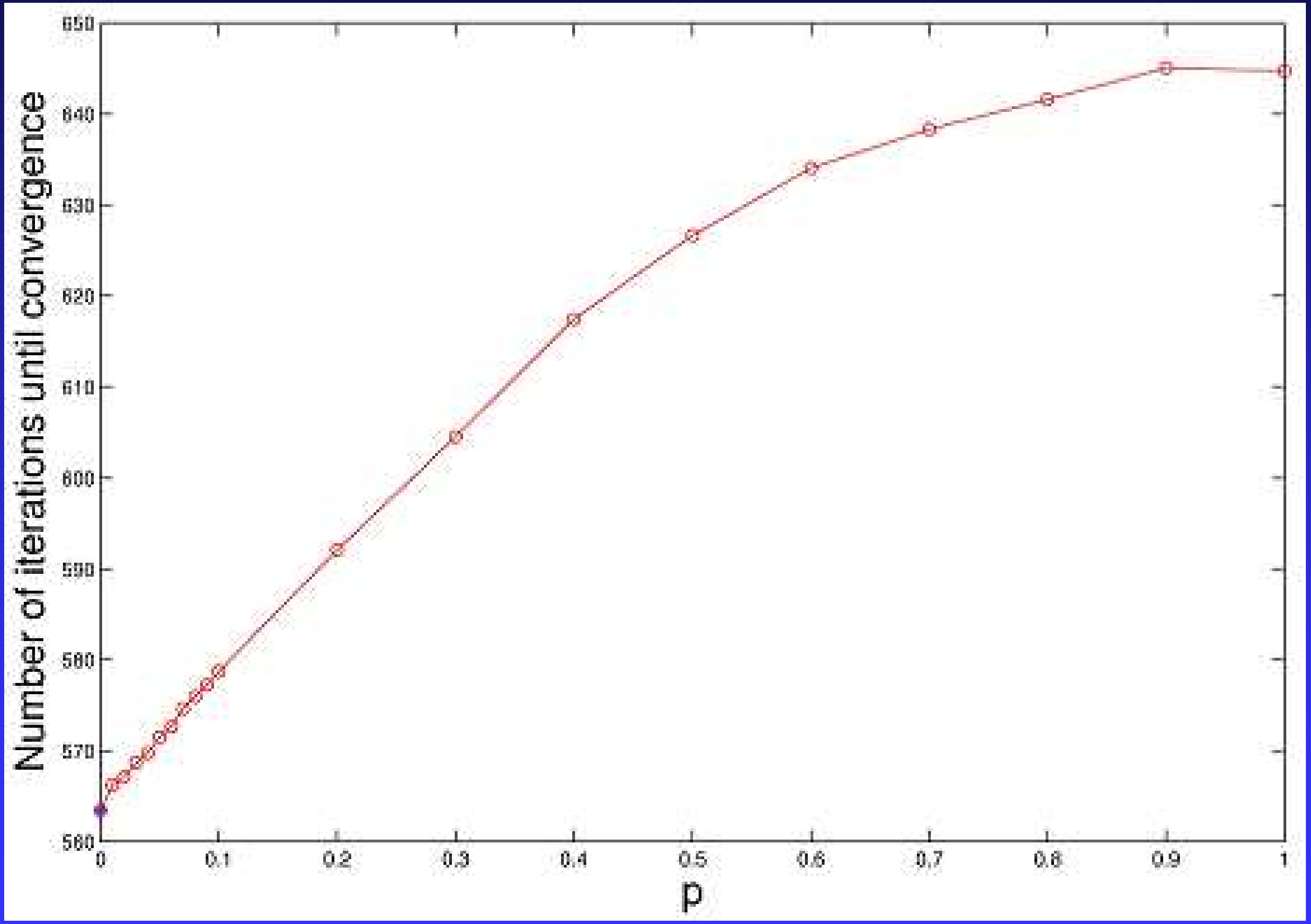
Experiments. Arc rewiring (max)



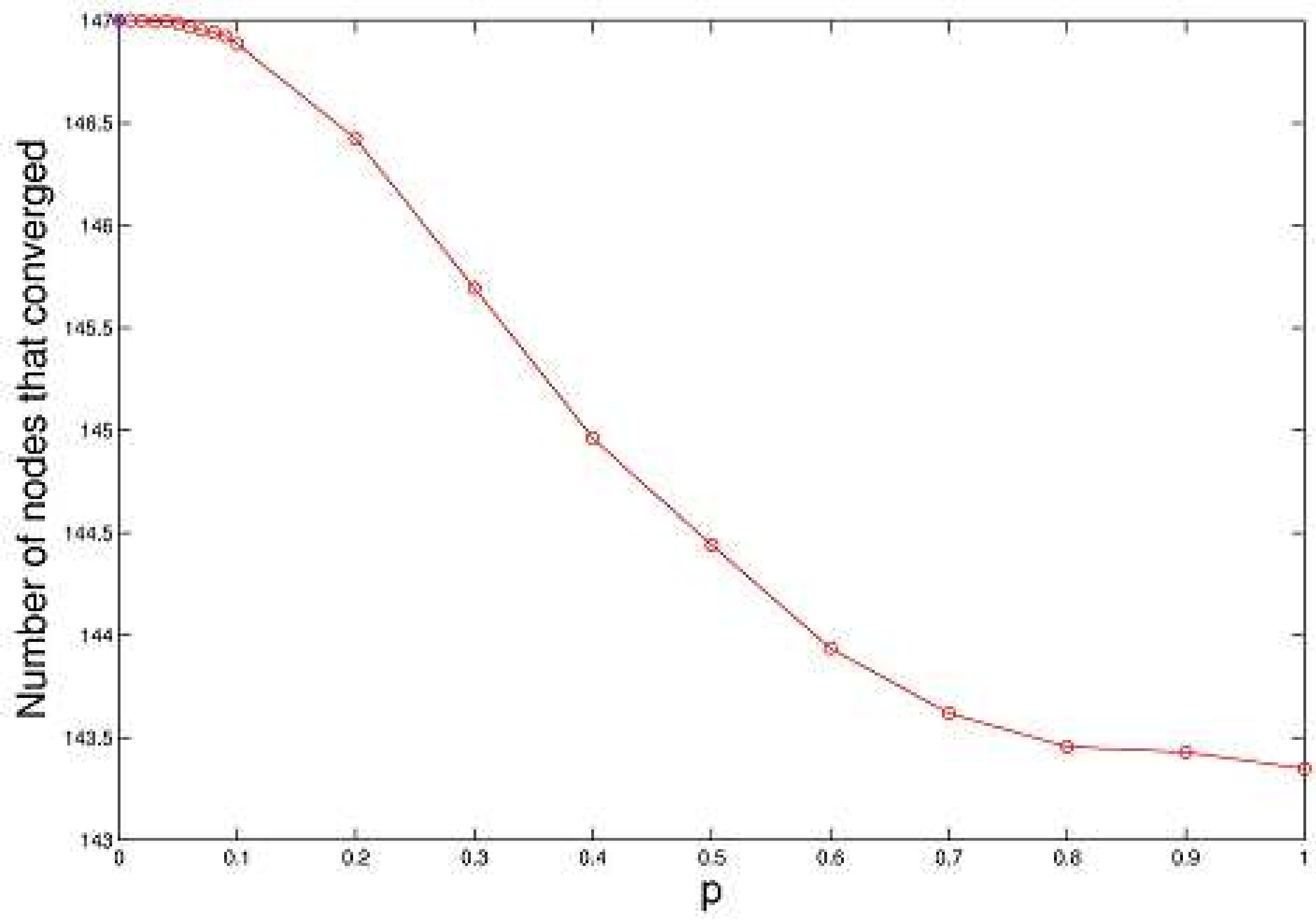
Experiments. Arc rewiring (max)



Experiments. Arc rewiring (sum)

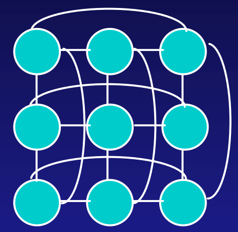


Experiments. Arc rewiring (sum)

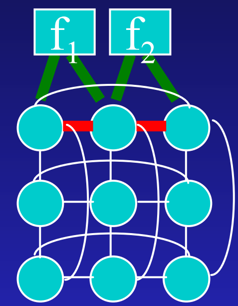


Experiments. Adding shortcuts

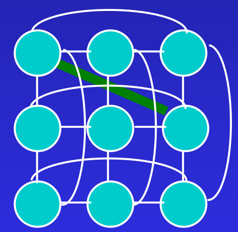
Starting from a grid (7x7)



50 sets of random functions for the grid



10 arcs will be added
Random, Max. dist, Min. dist.



50 instances

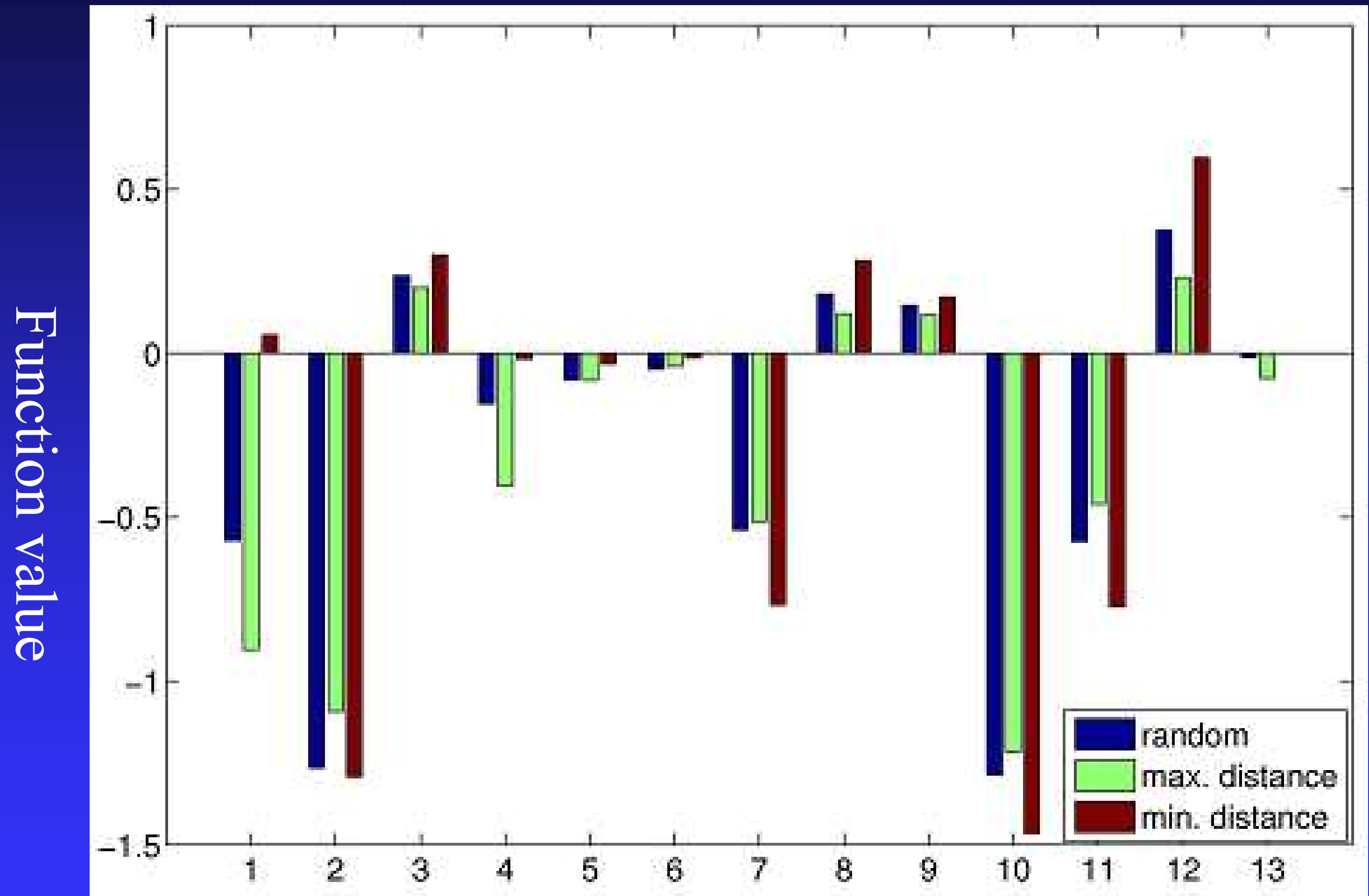
Sum-product and Max-product

Total: 15.000 runs

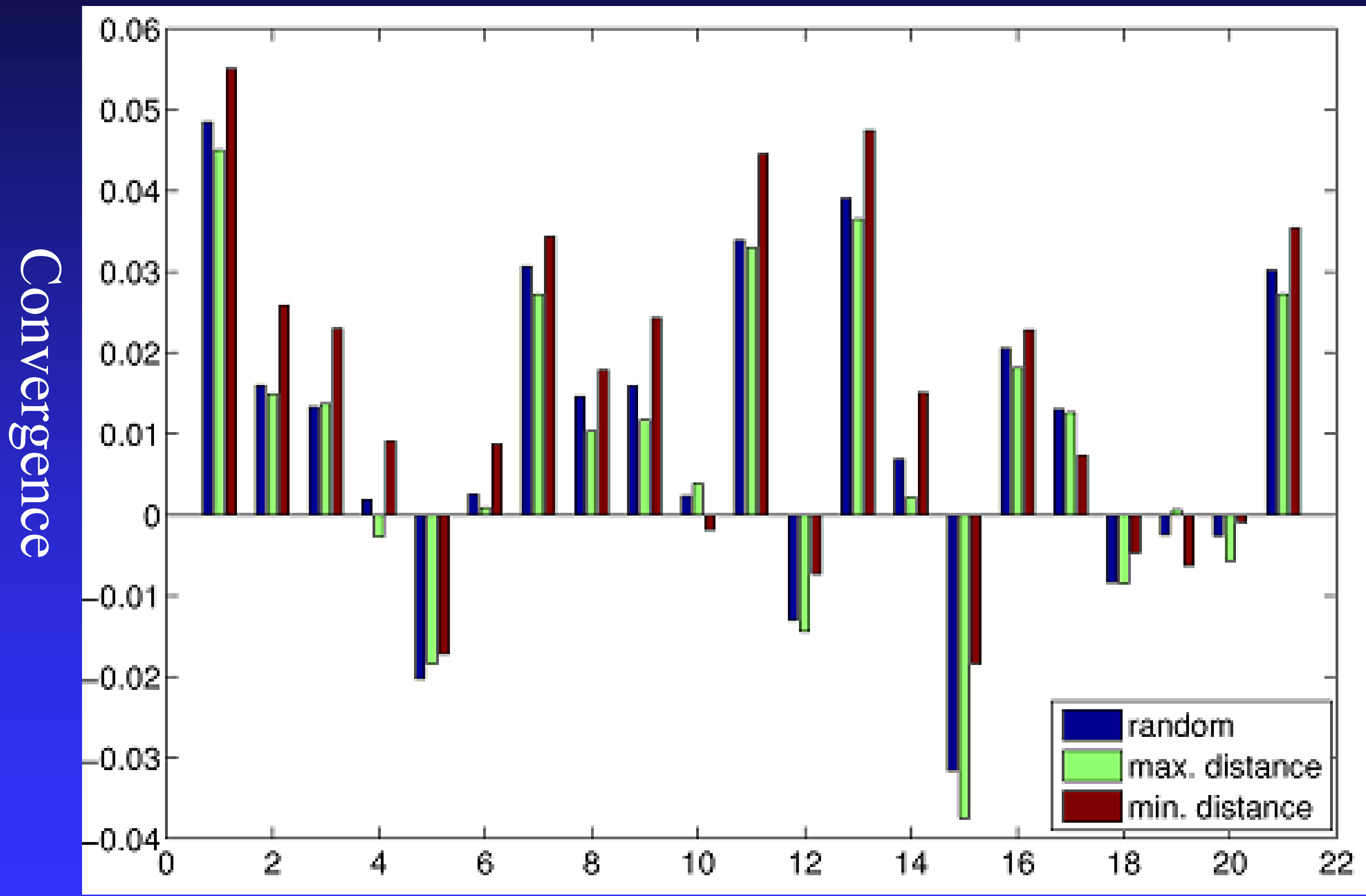
Experiments. Adding shortcuts (max)

- Compare to the value of the original grid
  - Better in 9, 9, and 10 of the instances
  - Equal in 22, 22, and 24 of the instances
  - We completed some statistical tests to check differences between the three methods

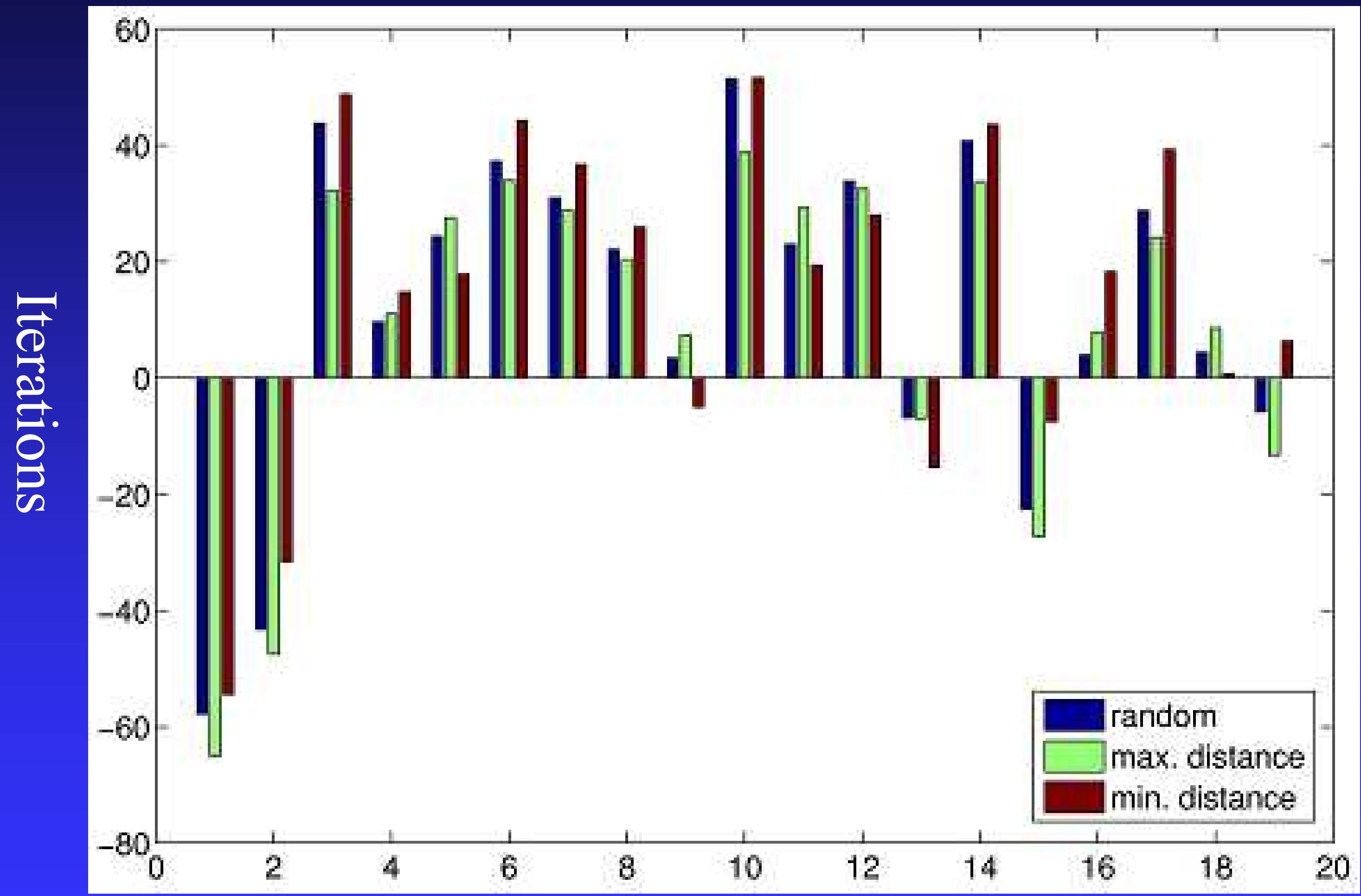
Experiments. Adding shortcuts (max)



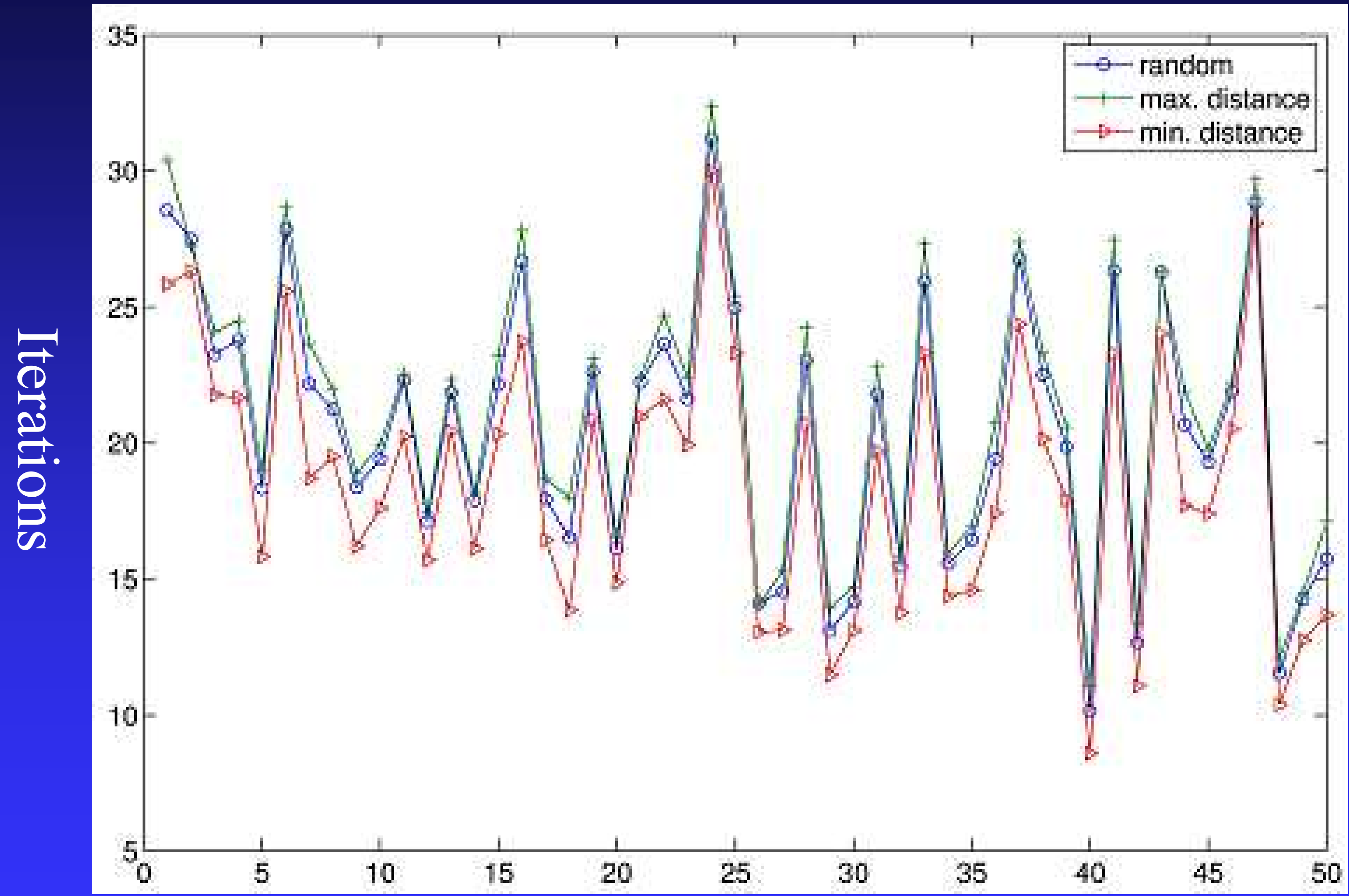
Experiments. Adding shortcuts (max)



Experiments. Adding shortcuts (max)



Experiments. Adding shortcuts (sum)



Conclusions

- Empirical analysis of LBP in different topologies
  - Rewiring
    - Finding optimal solutions is harder as p increases
  - Adding shortcuts
    - Changes dynamics of LBP
    - Min distance seems to be more promising
    - On-line method?

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Thank you !!

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