Complexity Results for Enumerating MPE and Partial MAP

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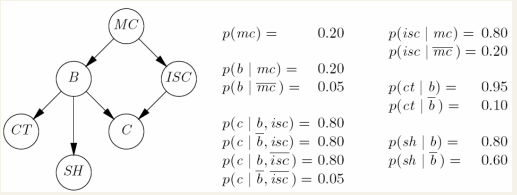
Enumerating MPE and Partial MAP

- **MPE/Partial MAP problem**: what is the most likely joint value assignment to a set of variables given complete, respectively partial, evidence of the complement of that set?
  - **MPE Example**: Given evidence of *mc*, what is the most likely joint value assignment to the other variables in the network?
  - **Partial MAP Example**: Given evidence of *mc*, what is the most likely joint value assignment to *ST*, *SH*, and *C*?



Brain Tumor Network

- However, often we are not only interested in the most likely joint value assignment, but also the 2nd most likely etc. and we want to enumerate joint value assignments
  - medical example: prescribe medication that covers a **number of likely causes**
  - compare best with second best: **how good** is the best explanation?
  - how **sensitive** is the MPE to small changes in value assignments?
- For the Kth MPE problem, algorithms are known to find k-th best explanations
- Problem is NP-hard, but exact complexity is unknown
- Even less information is available for Kth Partial MAP
- **Our contribution: complexity results for Kth MPE and Kth Partial MAP**

Complexity Theory

- Many problems related to probabilistic networks are NP-hard in general
- More specific complexity results are known for a variety of problems
- **NP**-complete: e.g. Most Probable Explanation
- **PP**-complete: e.g. Inference
- **NP^{PP}**-complete: e.g. Partial MAP, Parameter Tuning
- The class **P^{PP}** consists of problems, solvable by a *deterministic* Turing Machine, with access to an oracle for problems in PP (like Inference).
- **P^{PP}** is a less known, but very powerful class: it contains the entire polynomial hierarchy PH.
- The class **P$^{PP^{PP}}$** augments **P^{PP}** with an additional PP oracle.

'quick reference card'

Class	Complete problem	Intuition
NP	3SAT	Existence of a solution
P^{NP}	LexicalSAT	Property of the best solution
PP	MajSAT	Majority of solutions
P^{PP}	MidSAT	Property of the middle solution
NP^{PP}	E-MajSAT	Existence & Counting
P$^{PP^{PP}}$	Mid-MajSAT	Property of the middle solution & Counting

Discussion

- Finding the *k-th most likely* joint value assignment is considerately **harder** than finding the *most likely* joint value assignment
- Concrete: P^{PP} includes the entire **polynomial hierarchy** (Toda, 91)
- Kth Partial MAP is the ***first*** 'real world' problem proven to be complete for the (exotic) complexity class P$^{PP^{PP}}$
- However, many problems dealing with uncertainty combine some sort of **enumeration** and **stochastic reasoning**
- **E.g. planning, scheduling, other graphical models**

Results and Further Research

- **Main result:**
  Finding the K-th MPE is P^{PP}-complete, and finding the K-th Partial MAP is P$^{PP^{PP}}$-complete, making these problems considerately harder than MPE and Partial MAP, respectively.
- **Future research:**
  Parameterized problems variants: are Kth MPE and Kth Partial MAP *fixed parameter tractable?*

References

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